Canonical Approach to String Theory in Massive Background Fields

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Abstract

A method of constructing a canonical gauge invariant quantum formulation for a non-gauge classical theory depending on a set of parameters is advanced and then applied to the theory of closed bosonic string interacting with massive background fields. Choosing an ordering prescription and developing a suitable regularization technique we calculate quantum guage algebra up to linear order in background fields. Requirement of closure for the algebra leads to equations of motion for massive background fields which appear to be consistent with the structure of string spectrum.

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1 Introduction

$\sigma-$model approach still remains an important method of string theory investigation allowing one to derive string amplitudes as expectation values of corresponding vertex operators [1] (see also the reviews [2]). The main result received within this approach by means of perturbative path integral methods is that quantum conformal invariance condition leads to effective equations of motion for massless background fields. This condition appears as independence of quantum effective action on the conformal factor of two-dimensional metric or as vanishing of renormalized operator of the energy-momentum trace.

It should be noted that according to the prescription [3] generally accepted in functional approaches to string theory dynamical variables are treated in different ways. Namely, functional integration is carried out only over string coordinates $X^{\mu}(\tau,\sigma)$ while components of two-dimensional metric $g_{ab}(\tau,\sigma)$ are considered as external fields. Then one demands the result of such an integration to be independent on the conformal factor and the integrand over $g_{ab}(\tau,\sigma)$ reduces to finite dimensional integration over parameters specifying string world sheet topologies. This prescription differs from the standard field theory rules when functional integral is calculated over every variable independently [1].

This approach can be as well applied to string theory interacting with massive background fields which is not classically conformal invariant [8]. One demands that operator of the energy-momentum vanish no matter whether the corresponding classical action is conformal invariant or not. As was shown in [9, 10] it gives rise to effective equations of motion for massive background fields (See also [11, 12]). Unfortunately, in the case of closed string theory covariant approaches did not reproduce the full set of correct linear equations of motion for massive background fields. Namely, the tracelessness condition on massive tensor fields has been obtained neither in standard covariant perturbation approaches [9] nor within the formalism of exact renormalization group [12]. So there exists a problem of deriving the correct equations for massive background fields in framework of $\sigma-$model approach.

Moreover, from general point of view the requirement of quantum conformal invariance of string theory with massive background fields means that a non-gauge classical theory depending on a set of parameters is used for constructing of a quantum theory that is gauge invariant under some special values of the parameters. Such a situation occurs in string theory if interaction with massless dilaton, tachyon or any other massive field is turned on and it leads to a general problem in the theory of quantization.

The most natural and consistent approach to construction of quantum models in modern theoretical physics is the procedure of canonical quantization. We consider this approach to be the only completely consistent method for constructing quantum theories and so every step of any quantization procedure should be justified by

[1] In string theory this independent integration would lead to appearance at the quantum level of an extra degree of freedom connected with two-dimensional gravity [4].

an appropriate prescription within canonical formulation. So the general problem arising from string theories is how to describe in terms of canonical quantization construction of gauge invariant quantum theory starting with a classical theory without this invariance.

The most general realization of this procedure ensuring unitarity at the quantum level and consistency of theory symmetries and dynamics is BFV method [5, 6, 7]. Due to this method one should construct hamiltonian formulation of classical theory, find out all first class constraints and calculate algebra of their Poisson brackets. Then one defines fermionic functional Ω generating algebra of gauge transformations and bosonic functional H containing information of theory dynamics. Quantum theory is consistent provided that the operator $\hat{\Omega}$ is nilpotent and conserved in time. The corresponding analysis was carried out in [13, 14] for free bosonic string and in [15, 16] for bosonic string coupled to massless background fields.

In the case of string theory interacting with massive background fields components of two-dimensional metric should be treated as external fields, otherwise classical equations of motion would be inconsistent. As a consequence, classical gauge symmetries are absent and it is impossible to construct classical gauge functional Ω. In this paper we propose a prescription allowing for some models to construct quantum operator $\hat{\Omega}$ starting with a classical theory without first class constraints. Quantum theory is gauge invariant if there exist values of theory parameters providing nilpotency and conservation of operator $\hat{\Omega}$. Then to illustrate how the prescription works we apply it to the theory of closed bosonic string coupled to massive background fields and show that correct linear equations of motion are produced.

Calculation of quantum gauge algebra for the string interacting with background fields is not trivial for two reasons. First of all, the ordering prescription usually used in free string theory when all the ambiguity reduces to two constants $\alpha(0)$, $\beta(0)$ [20] is not enough in the case of string interacting with background fields. To deal with this issue we use a specific operators ordering with an ambiguity that was absent in the free string theory but is relevant in our case. Second, in presence of background fields quantum deformation of the conformal algebra contains divergencies which should be regularized in an appropriate way. In order to solve this problem we propose a kind of analytical regularization which depends on a single parameter and ensures finiteness of all contractions of the fundamental operators.

The paper is organized as follows. In Sec. 2 we describe a general method of constructing a gauge invariant canonical quantum formulation for a non-gauge classical theory. Sec. 3 contains description of the theory of closed bosonic string coupled to background fields of tachyon and of the first massive level. We build hamiltonian formulation for the model and show that it can be studied using the general method of the previous section. Sec. 4 is devoted to calculation of conformal algebra. Conclusion contains summary and outlooks.

2 From a non-gauge classical theory to a gauge invariant quantum theory

Consider a system described by a hamiltonian

$$H = H_0(a) + \lambda^\alpha T_\alpha(a) \tag{1}$$

where $H_0(a) = H_0(q, p, a)$, $T_\alpha(a) = T_\alpha(q, p, a)$ and q, p are canonically conjugated dynamical variables; $a = a_i$ and λ^α are external parameters of the theory.

We suppose that $T_\alpha(a)$ are some functions of the form

$$T_\alpha(a) = T_\alpha^{(0)}(a) + T_\alpha^{(1)}(a) \tag{2}$$

and closed algebra in terms of Poisson brackets is formed by $T_\alpha^{(0)}(a)$, not by $T_\alpha(a)$:

$$\begin{aligned} \{T_\alpha^{(0)}(a), T_\beta^{(0)}(a)\} &= T_\gamma^{(0)}(a) U_{\alpha\beta}^\gamma(a) \\ \{H_0(a), T_\alpha^{(0)}(a)\} &= T_\gamma^{(0)}(a) V_\alpha^\gamma(a) \end{aligned} \tag{3}$$

Such a situation may occur, for example, if $T_\alpha^{(0)}(a)$ correspond to a free gauge invariant theory and $T_\alpha^{(1)}(a)$ describe a perturbation spoiling gauge invariance.

At the quantum level both the algebras of $T_\alpha^{(0)}(a)$ and $T_\alpha(a)$ are not closed in general case

$$\begin{aligned} [T_\alpha(a), T_\beta(a)] &= i\hbar(T_\gamma(a) U_{\alpha\beta}^\gamma(a) + A_{\alpha\beta}(a)), \\ [H_0(a), T_\alpha(a)] &= i\hbar(T_\gamma(a) V_\alpha^\gamma(a) + A_\alpha(a)), \end{aligned} \tag{4}$$

and operators $A_{\alpha\beta}$, A_α do not vanish in the limit $\hbar \to 0$ due to absence of classical gauge invariance.

The main step of our approach consist in defining of the quantum operators Ω and H as if the functions $T_\alpha(a)$ were first class constraints:

$$\begin{aligned} \Omega &= c^\alpha T_\alpha(a) - \frac{1}{2} U_{\alpha\beta}^\gamma(a) : \mathcal{P}_\gamma c^\alpha c^\beta : \\ H &= H_0(a) + V_\alpha^\gamma(a) : \mathcal{P}_\gamma c^\alpha : \end{aligned} \tag{5}$$

where : : stands for some ordering of ghost fields. Square of such an operator Ω and its time derivative take the form

$$\begin{aligned} \Omega^2 &= \frac{1}{2} i\hbar (A_{\alpha\beta}(a) + G_{\alpha\beta}(a)) : c^\alpha c^\beta : \\ \frac{d\Omega}{dt} &= \frac{\partial \Omega}{\partial t} + [H, \Omega] = \\ &= \left(\frac{\partial T_\alpha(a)}{\partial t} - A_\alpha(a) - G_\alpha(a) \right) c^\alpha - \frac{1}{2} \frac{\partial U_{\alpha\beta}^\gamma(a)}{\partial t} : \mathcal{P}_\gamma c^\alpha c^\beta : \end{aligned} \tag{6}$$

where $G_{\alpha\beta}(a)$, $G_\alpha(a)$ are possible quantum contributions of ghosts.

It is natural to suppose that every operator of the theory can be decomposed in a linear combination of an irreducible set of independent operators $O_M(q,p)$:

$$\begin{aligned} &A_{\alpha\beta}(a) + G_{\alpha\beta}(a) = E^M_{\alpha\beta}(a) O_M(q,p), \\ &\frac{\partial T_\alpha(a)}{\partial t} - A_\alpha(a) - G_\alpha(a) = E^M_\alpha(a) O_M(q,p), \end{aligned} \tag{7}$$

$E^M_{\alpha\beta}(a)$, $E^M_\alpha(a)$ being some $c-$valued functions of the parameters a.

In general case $\Omega^2 \neq 0$ and $d\Omega/dt \neq 0$. However, if equations

$$E^M_{\alpha\beta}(a) = 0, \quad E^M_\alpha(a) = 0, \quad \frac{\partial U^\gamma_{\alpha\beta}(a)}{\partial t} = 0, \tag{8}$$

have some solutions $a_i = a_i^{(0)}$ then operator Ω is nilpotent and conserved for these specific values of parameters and hence the quantum theory is gauge invariant. Thus, there exists a possibility to construct quantum theory with given gauge invariance that is absent at the classical level.

It is important that eqs.(8) are not anomaly cancellation conditions because an anomaly represents breaking of classical symmetries at the quantum level. In the theory under consideration classical symmetries are absent and eqs.(8) express conditions of quantum symmetries existence.

In specific models eqs.(8) may have no solutions at all or, conversely, be fulfilled identically. An example of the latter possibility was described in [17].

3 String in massive background fields

As an example where the described procedure really works and leads to eqs.(8) with non-trivial solutions for parameters a we consider closed bosonic string theory coupled with background fields of tachyon and of the first massive level. We will restrict ourselves by linear approximation in background fields. [2] It will be enough to establish consistency of background fields dynamics with structure of the corresponding massive level in string spectrum.

The theory is described by the classical action

$$\begin{aligned} S \;=\; & -\frac{1}{2\pi\alpha'}\int d^2\sigma\,\sqrt{-g}\{\frac{1}{2}g^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu} + Q(X) \\ & + g^{ab}g^{cd}\partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \partial_d X^\kappa F^1_{\mu\nu\lambda\kappa}(X) \\ & + g^{ab}\varepsilon^{cd}\partial_a X^\mu \partial_b X^\nu \partial_c X^\lambda \partial_d X^\kappa F^2_{\mu\nu\lambda\kappa}(X) \\ & + \alpha' R^{(2)} g^{ab}\partial_a X^\mu \partial_b X^\nu W^1_{\mu\nu}(X) + \alpha' R^{(2)}\varepsilon^{ab}\partial_a X^\mu \partial_b X^\nu W^2_{\mu\nu}(X) \\ & + \alpha'^2 R^{(2)} R^{(2)} C(X)\}, \end{aligned} \tag{9}$$

[2] An adequate treatment of non-linear (interaction) terms is known to demand non-perturbative methods [18]. Our aim here is just to illustrate possibilities of the general prescription given in the previous section.

$\sigma^a = (\tau, \sigma)$ are coordinates on string world sheet, $R^{(2)}$ is scalar curvature of two-dimensional metric g^{ab}, $\eta_{\mu\nu}$ is Minkowski metric of $D-$dimensional spacetime, $Q(X)$ is tachyonic field and F, W, C are background fields of the first massive level in string spectrum. As was shown in [9] all other possible terms with four two-dimensional derivatives in classical action are not essential and string interacts with background fields of the first massive level only by means of the terms presented in (9).

If components of two-dimensional metric g_{ab} were considered as independent dynamical variables then the corresponding classical equations of motion would be fulfilled only for vanishing background fields:

$$
\begin{aligned}
\frac{2\pi}{\sqrt{-g}} g_{ab} \frac{\delta S}{\delta g_{ab}} \quad = \quad & -(1/\alpha')\sqrt{-g} Q(X) \\
& + g^{ab} g^{cd} \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda \partial_d x^\kappa (-(1/\alpha') F^1_{\mu\nu\lambda\kappa} + \partial_\mu \partial_\nu W^1_{\lambda\kappa}) \\
& + g^{ab} \varepsilon^{cd} \partial_a x^\mu \partial_b x^\nu \partial_c x^\lambda \partial_d x^\kappa (-(1/\alpha') F^2_{\mu\nu\lambda\kappa} + \partial_\mu \partial_\nu W^2_{\lambda\kappa}) \\
& + g^{ab} D^2 x^\mu \partial_a x^\nu \partial_b x^\lambda \partial_\mu W^1_{\nu\lambda} + g^{ac} g^{bd} D_a \partial_b x^\mu \partial_c x^\nu \partial_d x^\lambda 4 \partial_\lambda W^1_{\mu\nu} \\
& + g^{ac} \varepsilon^{bd} D_a \partial_b x^\mu \partial_c x^{[\nu} \partial_d x^{\lambda]} 2 (\partial_\mu W^2_{\nu\lambda} + \partial_\nu W^2_{\mu\lambda} - \partial_\lambda W^2_{\mu\nu}) \\
& + g^{ac} \varepsilon^{bd} D_a \partial_b x^\mu \partial_c x^{(\nu} \partial_d x^{\lambda)} 2 (\partial_\nu W^2_{\mu\lambda} + \partial_\lambda W^2_{\mu\nu}) \\
& + g^{ac} g^{bd} D_a \partial_b x^\mu D_c \partial_d x^\nu 2 W^1_{\mu\nu} + g^{ac} \varepsilon^{bd} D_a \partial_b x^\mu D_c \partial_d x^\nu 2 W^2_{\mu\nu} \\
& + g^{ab} D_a D^2 x^\mu \partial_b x^\nu 2 W^1_{\mu\nu} + \varepsilon^{ab} D_a D^2 x^\mu \partial_b x^\nu 2 W^2_{\mu\nu} + \\
& + R^{(2)} g^{ab} \partial_a x^\mu \partial_b x^\nu 2 (-W^1_{\mu\nu} + \alpha' \partial_\mu \partial_\nu C) \\
& + R^{(2)} \varepsilon^{ab} \partial_a x^\mu \partial_b x^\nu (-2 W^2_{\mu\nu}) + g^{ab} \partial_a x^\mu \partial_b R^{(2)} 4 \alpha' \partial_\mu C \\
& + R^{(2)} D^2 x^\mu 2 \alpha' \partial_\mu C + D^2 R^{(2)} \, 2\alpha' C + R^{(2)} R^{(2)} (-\alpha' C) = 0. \quad (10)
\end{aligned}
$$

Analogous situation arises in the theory of string interacting with massless dilaton field [15]. Therefore to construct the theory with non-trivial massive background fields we have to consider the components g_{ab} as external fields. Such a treatment corresponds to covariant methods where functional integral is calculated from the very beginning only over X^μ variables.

After the standard parametrization of metric

$$
g_{ab} \quad = \quad e^\gamma \begin{pmatrix} \lambda_1^2 - \lambda_0^2 & \lambda_1 \\ \lambda_1 & 1 \end{pmatrix} \tag{11}
$$

the hamiltonian in linear approximation in background fields takes the form

$$
H = \int d\sigma \, (p_\mu \dot{x}^\mu - L) = \int d\sigma \, (\lambda_0 T_0 + \lambda_1 T_1), \tag{12}
$$

where

$$
\begin{aligned}
& T_0 = T_0^{(0)} + T_0^{(1)}, \\
& T_0^{(0)} = \frac{1}{2} \left(2\pi\alpha' P^2 + (1/2\pi\alpha) X'^2 \right),
\end{aligned}
$$

$$
\begin{aligned}
T_0^{(1)} = \frac{1}{2\pi}\Big(& (1/\alpha')e^{\gamma}Q(X) + (1/\alpha')e^{-\gamma}\Big[(2\pi\alpha')^4 P^\mu P^\nu P^\lambda P^\kappa \\
& \qquad - 2(2\pi\alpha')^2 P^\mu P^\nu X'^\lambda X'^\kappa + X'^\mu X'^\nu X'^\lambda X'^\kappa\Big] F^1_{\mu\nu\lambda\kappa}(X) \\
& + (2/\alpha')e^{-\gamma}\Big[-(2\pi\alpha')^3 P^\mu P^\nu P^\lambda X'^\kappa + 2\pi\alpha' X'^\mu X'^\nu P^\lambda X'^\kappa\Big] F^2_{\mu\nu\lambda\kappa}(X) \\
& + R^{(2)}\Big[-(2\pi\alpha')^2 P^\mu P^\nu + X'^\mu X'^\nu\Big] W^1_{\mu\nu}(X) + 4\pi\alpha' R^{(2)} P^\mu X'^\nu W^2_{\mu\nu}(X) \\
& + \alpha' e^{\gamma} R^{(2)} R^{(2)} C(X)\Big), \\
T_1 = T_1^{(0)} = {} & P_\mu X'^\mu. \qquad (13)
\end{aligned}
$$

$X'^\mu = \partial X^\mu/\partial\sigma$, and P_μ are momenta canonically conjugated to X^μ. $T_0^{(0)}$ and $T_1^{(0)}$ represent constraints of free string theory and form closed algebra in terms of Poisson brackets. λ_0 and λ_1 play the role of external fields and so T_0 and T_1 cannot be considered as constraints of classical theory. In free string theory conditions $T_0^{(0)} = 0$, $T_1^{(0)} = 0$ result from conservation of canonical momenta conjugated to λ_0 and λ_1. According to our prescription in string theory with massive background fields λ_0 and λ_1 can not be considered as dynamical variables, there are no corresponding momenta and conditions of their conservation do not appear.

The role of parameters a in the theory under consideration is played by background fields Q, F, W, C and conformal factor of two-dimensional metric $\gamma(\tau,\sigma)$. The theory (12) is of the type (1) with $H_0 = 0$, structural constants of classical algebra being independent on time. It means that the condition of conservation for the operator Ω (6) will be satisfied if T_0 does not depend on time explicitly, that is

$$
\frac{\partial\gamma(\tau,\sigma)}{\partial\tau} = 0, \qquad \frac{\partial R^{(2)}(\tau,\sigma)}{\partial\tau} = 0. \qquad (14)
$$

To derive the first condition (8) one has to construct quantum algebra of operators T_0, T_1.

4 Quantum guage algebra and effective equations of motion

To carry out the calculation of the quantum algebra we first of all introduce the following notations for so called right and left moving Fubini fields

$$
Y^{\pm\mu} = 2\pi\alpha' P^\mu \mp X'^\mu \qquad (15)
$$

and turn to the discrete set of operators:

$$
\begin{aligned}
L_n &= \int_0^{2\pi} d\sigma\, e^{-in\sigma}\frac{1}{2}(T_0 - T_1) = L_n^{(0)} + K_n, \\
\bar{L}_n &= \int_0^{2\pi} d\sigma\, e^{in\sigma}\frac{1}{2}(T_0 + T_1) = \bar{L}_n^{(0)} - K_{-n}. \qquad (16)
\end{aligned}
$$

Here operators

$$
\begin{aligned}
L_n^{(0)} &= \int_0^{2\pi} d\sigma\, e^{-in\sigma}\frac{1}{2}(T_0^{(0)} - T_1^{(0)}) = \int_0^{2\pi} d\sigma\, e^{-in\sigma}\frac{1}{8\pi\alpha'}Y^{+\mu}Y^{+\nu}\eta_{\mu\nu}, \\
\bar{L}_n^{(0)} &= \int_0^{2\pi} d\sigma\, e^{in\sigma}\frac{1}{2}(T_0^{(0)} + T_1^{(0)}) = \int_0^{2\pi} d\sigma\, e^{in\sigma}\frac{1}{8\pi\alpha'}Y^{-\mu}Y^{-\nu}\eta_{\mu\nu}
\end{aligned}
\tag{17}
$$

represent the set of constraints of the free string theory and form the well known quantum algebra

$$
\begin{aligned}
[L_n^{(0)}, L_m^{(0)}] &= \hbar(n-m)L_{n+m}^{(0)} + \hbar^2\delta_{n,-m}\left(\frac{D}{12}n(n^2-1) + 2\alpha(0)n\right), \\
[\bar{L}_n^{(0)}, \bar{L}_m^{(0)}] &= \hbar(n-m)\bar{L}_{n+m}^{(0)} + \hbar^2\delta_{n,-m}\left(\frac{D}{12}n(n^2-1) + 2\beta(0)n\right), \\
[L_n^{(0)}, \bar{L}_m^{(0)}] &= 0,
\end{aligned}
\tag{18}
$$

$\alpha(0)$, $\beta(0)$ are constants fixing the operators ordering ambiguity.

The operators K_n are contributions linear in background fields and have the form

$$
\begin{aligned}
K_n = \frac{1}{4\pi\alpha'}\int_0^{2\pi} d\sigma\, e^{-in\sigma}\Big\{ & e^{\gamma}Q(X) + e^{-\gamma}Y^{+\mu}Y^{+\nu}Y^{-\lambda}Y^{-\kappa}F_{\mu\nu\lambda\kappa}(X) \\
& + \alpha' R^{(2)}Y^{+\mu}Y^{-\nu}W_{\mu\nu}(X) + (\alpha')^2 e^{\gamma}R^{(2)}R^{(2)}C(X)\Big\}
\end{aligned}
\tag{19}
$$

where we introduced the following notations:

$$
\begin{aligned}
F_{\mu\nu\lambda\kappa} &= 2F^1_{\mu\lambda\nu\kappa} + 2F^1_{\mu\kappa\nu\lambda} - 2F^2_{\mu\lambda\nu\kappa} - 2F^2_{\nu\kappa\mu\lambda}, \\
W_{\mu\nu} &= -W^1_{\mu\nu} + W^2_{\mu\nu}.
\end{aligned}
\tag{20}
$$

As periodic functions of σ the canonical operators of string coordinates and momenta have standard Fourier expansions:

$$
\begin{aligned}
X^\mu &= \frac{1}{\sqrt{2\pi}}x_0^\mu(\tau) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{1}{n}(\alpha_n^\mu(\tau)z^n + \bar{\alpha}_n^\mu(\tau)z^{-n}), \quad z = e^{i\sigma}, \\
P^\mu &= \frac{1}{\sqrt{2\pi\alpha'}}p_0^\mu(\tau) + \frac{1}{2\pi\sqrt{2\alpha'}}\sum_{n\neq 0}(\alpha_n^\mu(\tau)z^n + \bar{\alpha}_n^\mu(\tau)z^{-n})
\end{aligned}
\tag{21}
$$

where operators of zero modes x_0^μ, p_0^μ and oscillating ones α_n^μ, $\bar{\alpha}_n^\mu$ satisfy the following commutation relations (we have set $\hbar = 1$ for the rest of the paper):

$$
[x_0^\mu, p_{0\nu}] = i\delta_\nu^\mu,\ [\alpha_m^\mu, \alpha_n^\nu] = [\bar{\alpha}_m^\mu, \bar{\alpha}_n^\nu] = m\delta_{m,-n}\eta^{\mu\nu}. \tag{22}
$$

The most general ordering in terms of the operators x_0^μ, p_0^μ, α_n^μ, $\bar{\alpha}_n^\mu$ can be written down as

$$
\begin{aligned}
O(\alpha_m^\mu\alpha_n^\nu) &= (1 - c_m^\mu)\alpha_m^\mu\alpha_n^\nu + c_m^\mu\alpha_n^\nu\alpha_m^\mu = \alpha_m^\mu\alpha_n^\nu - mc_m^\mu\delta_{m,-n}\eta^{\mu\nu}, \\
O(\bar{\alpha}_m^\mu\bar{\alpha}_n^\nu) &= (1 - \bar{c}_m^\mu)\bar{\alpha}_m^\mu\bar{\alpha}_n^\nu + \bar{c}_m^\mu\bar{\alpha}_n^\nu\bar{\alpha}_m^\mu = \bar{\alpha}_m^\mu\bar{\alpha}_n^\nu - m\bar{c}_m^\mu\delta_{m,-n}\eta^{\mu\nu}, \\
O(x_0^\mu p_{\nu 0}) &= (1 - c_0^\mu)x_0^\mu p_{\nu 0} + c_0^\mu p_{\nu 0}x_0^\mu = x_0^\mu p_{\nu 0} - ic_0^\mu\delta_\nu^\mu.
\end{aligned}
\tag{23}
$$

Here c^μ_m, $\bar{c}^\mu_m$, c^μ_0 are some constant parameters defining an ordering type and there are no summations over indices m and μ.

As one can commute operators within the sign of ordering

$$O(\alpha^\mu_m \alpha^\mu_{-m}) = O(\alpha^\mu_{-m}\alpha^\mu_m) \tag{24}$$

then there exist the following relation for parameters c^μ_m:

$$c^\mu_{-m} + c^\mu_m = 1 \tag{25}$$

and the analogous one for $\bar{c}^\mu_m$. It means that operators ordering is fixed up completely by choosing values for the infinite set of parameters c^μ_m, $\bar{c}^\mu_m$, c^μ_0, $m > 0$. For example, the special case of Wick ordering (that is such an ordering when all the creation operators α_n, $\bar{\alpha}_n$, $n < 0$ stand to the right of all the annihilating operators α_n, $\bar{\alpha}_n$, $n > 0$) is defined by the choosing $c^\mu_n = \theta(n)$:

$$:\alpha^\mu_m \alpha^\nu_n := \alpha^\mu_m \alpha^\nu_n - m\theta(m)\delta_{m,-n}\eta^{\mu\nu}. \tag{26}$$

Contractions of fundamental operators have the form:

$$\underbracket{\alpha^\mu_m \alpha^\nu_n} = \alpha^\mu_m \alpha^\nu_n - O(\alpha^\mu_m \alpha^\nu_n) = m c^\mu_m \delta_{m,-n}\eta^{\mu\nu},$$

$$\underbracket{\bar{\alpha}^\mu_m \bar{\alpha}^\nu_n} = m\bar{c}^\mu_m \delta_{m,-n}\eta^{\mu\nu}, \quad \underbracket{x^\mu_0 p_{0\nu}} = i c^\mu_0 \delta^\mu_\nu, \quad \underbracket{p_{0\nu} x^\mu_0} = i(c^\mu_0 - 1)\delta^\mu_\nu. \tag{27}$$

In the case of free string theory the whole ordering ambiguity of constraints operators reduces to the choice of two parameters in the following way. Operators $L^{(0)}_n$, $\bar{L}^{(0)}_n$ are quadratic in fundamental variables

$$L^{(0)}_n = \frac{1}{2}\sum_k \alpha^\mu_k \alpha^\nu_{n-k}\eta_{\mu\nu}, \quad \bar{L}^{(0)}_n = \frac{1}{2}\sum_k \bar{\alpha}^\mu_k \bar{\alpha}^\nu_{n-k}\eta_{\mu\nu} \tag{28}$$

and their arbitrary ordering can be expressed through the Wick one as follows:

$$\begin{aligned} O(L^{(0)}_n) &= \frac{1}{2}\sum_{k=-\infty}^{\infty} :\alpha^\mu_k \alpha^\nu_{n-k}\eta_{\mu\nu}: -\frac{1}{2}\sum_{k=-\infty}^{\infty}\sum_{\mu=0}^{D-1} k\delta_{n,0}c^\mu_k + \frac{1}{2}\sum_{k=-\infty}^{\infty}\sum_{\mu=0}^{D-1} k\delta_{n,0}\theta(k) \\ &= :L^{(0)}_n: -\alpha(0)\delta_{n,0}, \\ O(\bar{L}^{(0)}_n) &= :\bar{L}^{(0)}_n: -\beta(0)\delta_{n,0}, \end{aligned} \tag{29}$$

where parameters $\alpha(0)$ and $\beta(0)$ are defined as

$$\alpha(0) = \sum_{k=1}^{\infty}\sum_{\mu=0}^{D-1} k(c^\mu_k - 1), \quad \beta(0) = \sum_{k=1}^{\infty}\sum_{\mu=0}^{D-1} k(\bar{c}^\mu_k - 1). \tag{30}$$

Any set of parameters c^μ_n, $\bar{c}^\mu_n$ leading to the same values of $\alpha(0)$ and $\beta(0)$ defines equivalent quantum free string theories. Parameters c^μ_0 do not enter the operators

$L_n^{(0)}$, $\bar{L}_n^{(0)}$ at all and so $\alpha(0)$ and $\beta(0)$ are the only ordering parameters whose values are relevant in free string theory.

In the case of string interacting with background fields the operators L_n, $\bar{L}_n$ are arbitrary functions of the operators x_0^μ, p_0^μ, α_n^μ, $\bar{\alpha}_n^\mu$ and one has to take into account the whole infinite set of various parameters c_n^μ, $\bar{c}_n^\mu$, c_0^μ. We will calculate the quantum algebra of operators L_n, $\bar{L}_n$ using ordering parameters of the following form

$$
\begin{aligned}
c_k^\mu - 1 &= \frac{\alpha(0)}{D}(1-\Lambda)^2\Lambda^{k-1}, \quad k>0, \quad |\Lambda|<1,\\
\bar{c}_k^\mu - 1 &= \frac{\beta(0)}{D}(1-\bar{\Lambda})^2\bar{\Lambda}^{k-1}, \quad k>0, \quad |\bar{\Lambda}|<1,\\
c_0^\mu &= \frac{1}{2},
\end{aligned}
\tag{31}
$$

where Λ, $\bar{\Lambda}$ are complex variables. This choice is not the most general one but it is suitable for our purposes because it both is consistent with the free string ordering prescription (30) and by means of parameters Λ, $\bar{\Lambda}$ describes an ordering ambiguity which was not relevant in the free case but is sufficient for the string in background fields.

In this class of ordering prescription the contractions of fundamental variables have the form

$$
\begin{aligned}
&\underbracket{X^\mu(z_1)Y^{+\nu}}(z_2) = \\
&\quad = i\alpha'\eta^{\mu\nu}\left[\frac{1}{2}+\sum_{n>0}\left(\frac{z_1}{z_2}\right)^n\right] + i\alpha'\eta^{\mu\nu}\frac{\alpha(0)(1-\Lambda)^2}{\Lambda D}\sum_{n>0}\left[\left(\frac{\Lambda z_1}{z_2}\right)^n+\left(\frac{\Lambda z_2}{z_1}\right)^n\right],\\
&\underbracket{X^\mu(z_1)Y^{-\nu}}(z_2) = \\
&\quad = -i\alpha'\eta^{\mu\nu}\left[\frac{1}{2}+\sum_{n>0}\left(\frac{z_2}{z_1}\right)^n\right] - i\alpha'\eta^{\mu\nu}\frac{\beta(0)(1-\bar{\Lambda})^2}{\bar{\Lambda} D}\sum_{n>0}\left[\left(\frac{\bar{\Lambda} z_1}{z_2}\right)^n+\left(\frac{\bar{\Lambda} z_2}{z_1}\right)^n\right],\\
&\underbracket{Y^{+\mu}(z_1)Y^{+\nu}}(z_2) = \\
&\quad = \alpha'\eta^{\mu\nu}\sum_{n>0}n\left(\frac{z_1}{z_2}\right)^n + 2\alpha'\eta^{\mu\nu}\frac{\alpha(0)(1-\Lambda)^2}{\Lambda D}\sum_{n>0}n\left[\left(\frac{\Lambda z_1}{z_2}\right)^n+\left(\frac{\Lambda z_2}{z_1}\right)^n\right],\\
&\underbracket{Y^{-\mu}(z_1)Y^{-\nu}}(z_2) = \\
&\quad = \alpha'\eta^{\mu\nu}\sum_{n>0}n\left(\frac{z_2}{z_1}\right)^n + 2\alpha'\eta^{\mu\nu}\frac{\beta(0)(1-\bar{\Lambda})^2}{\bar{\Lambda} D}\sum_{n>0}n\left[\left(\frac{\bar{\Lambda} z_1}{z_2}\right)^n+\left(\frac{\bar{\Lambda} z_2}{z_1}\right)^n\right],\\
&\underbracket{Y^{+\mu}(z_1)Y^{-\nu}}(z_2) = 0 \qquad .
\end{aligned}
\tag{32}
$$

All the contractions contain divergent series and represent generalized functions of their argument $(z_1 - z_2)$. Quantum contributions to the operator algebra contain

products of these contractions and so one should regularize them by introducing a parameter which would make the contractions well defined analytical functions.

We introduce this parameter ϵ ($Re\,\epsilon > 0$) in such a way that shifts arguments of divergent series into the region of their convergence:

$$\frac{z_1}{z_2} \longrightarrow e^{-\epsilon}\frac{z_1}{z_2}, \quad \frac{z_2}{z_1} \longrightarrow e^{-\epsilon}\frac{z_2}{z_1}.$$

Then all the contractions can be summed up to elementary analytical functions:

$$\begin{aligned}
&X^\mu(z_1)Y^{+\nu}(z_2) = \\
&\quad = i\alpha'\eta^{\mu\nu}\left[\frac{1}{2} + \frac{z_1 e^{-\epsilon}}{z_2 - z_1 e^{-\epsilon}}\right] + i\alpha'\eta^{\mu\nu}\frac{\alpha(0)(1-\Lambda)^2}{\Lambda D}\left[\frac{z_1\Lambda}{z_2 - z_1\Lambda} + \frac{z_2}{z_2 - z_1\Lambda^{-1}}\right], \\
&X^\mu(z_1)Y^{-\nu}(z_2) = \\
&\quad = -i\alpha'\eta^{\mu\nu}\left[\frac{1}{2} + \frac{z_1 e^{\epsilon}}{z_2 - z_1 e^{\epsilon}}\right] - i\alpha'\eta^{\mu\nu}\frac{\beta(0)(1-\bar\Lambda)^2}{\bar\Lambda D}\left[\frac{z_1\bar\Lambda}{z_2 - z_1\bar\Lambda} + \frac{z_2}{z_2 - z_1\bar\Lambda^{-1}}\right], \\
&Y^{+\mu}(z_1)Y^{+\nu}(z_2) = \\
&\quad = \alpha'\eta^{\mu\nu}\frac{z_1 z_2 e^{-\epsilon}}{(z_2 - z_1 e^{-\epsilon})^2} + 2\alpha'\eta^{\mu\nu}\frac{\alpha(0)(1-\Lambda)^2}{\Lambda D}\left[\frac{z_1 z_2\Lambda}{(z_2 - z_1\Lambda)^2} + \frac{z_2 z_1\Lambda^{-1}}{(z_2 - z_1\Lambda^{-1})^2}\right], \\
&Y^{-\mu}(z_1)Y^{-\nu}(z_2) = \\
&\quad = \alpha'\eta^{\mu\nu}\frac{z_1 z_2 e^{\epsilon}}{(z_2 - z_1 e^{\epsilon})^2} + 2\alpha'\eta^{\mu\nu}\frac{\beta(0)(1-\bar\Lambda)^2}{\bar\Lambda D}\left[\frac{z_1 z_2\bar\Lambda}{(z_2 - z_1\bar\Lambda)^2} + \frac{z_2 z_1\bar\Lambda^{-1}}{(z_2 - z_1\bar\Lambda^{-1})^2}\right]
\end{aligned} \tag{33}$$

Integrations in operators L_n, $\bar L_n$ go over the unit circle in complex plane of the variable $z = e^{i\sigma}$. Our regularization scheme leads to disappearing of the singular point $z_2 = z_1$ on the contour of integration and to appearing of additional singular points $z_2 = z_1 e^{-\epsilon}$, $z_2 = z_1 e^{\epsilon}$ lying out of the contour. Therefore, all the integrals over z_2 can be done by means of calculating residues in internal poles $z_2 = 0$, $z_2 = z_1 e^{-\epsilon}$, $z_2 = z_1\Lambda$, $z_2 = z_1\bar\Lambda$. The regularization parameter ϵ should be set equal to zero after doing all calculations.

Now we are ready to calculate commutators $[L_n^{(0)} + K_n, L_m^{(0)} + K_m]$. We are interested only in contributions linear in background fields and so can omit the commutator $[K_n, K_m]$ because each term in operators K_n depends on background fields. Operators $L_n^{(0)}$ are quadratic and so commutators $[K_n, L_m^{(0)}]$ contain quantum contributions only with products of two contractions.

The general expression for these commutators can be constructed with the use

of Wick theorem for product of two operators $A(\Gamma)$ and $B(\Gamma)$ [19]:

$$O(A(\Gamma))\,O(B(\Gamma)) = O\Big(\exp\Big(\underbrace{\Gamma_1^M\Gamma_2^N}\frac{\delta}{\delta\Gamma_1^M}\frac{\delta}{\delta\Gamma_2^N}\Big)A(\Gamma_1)B(\Gamma_2)\Big)\Big|_{\Gamma_1=\Gamma_2=\Gamma} \tag{34}$$

Set of canonical variables Γ in our case consists of the operators $X^\mu(\tau,z)$, $Y^{+\mu}(\tau,z)$, $Y^{-\mu}(\tau,z)$. After decomposition of canonical contractions into the sum of symmetrical and antisymmetrical parts

$$\underbrace{\Gamma^M\Gamma^N} = \underbrace{\Gamma^M\Gamma^N}_S + \underbrace{\Gamma^M\Gamma^N}_A \tag{35}$$

commutator of two operators according to the Wick theorem takes form of the following expansion in powers of contractions:

$$[A,B] = 2\underbrace{\Gamma^M\Gamma^N}_A\frac{\delta A}{\delta\Gamma^M}\frac{\delta B}{\delta\Gamma^N} + 4\underbrace{\Gamma^{M_1}\Gamma^{N_1}}_S\underbrace{\Gamma^{M_2}\Gamma^{N_2}}_A\frac{\delta^2 A}{\delta\Gamma^{M_1}\delta\Gamma^{M_2}}\frac{\delta^2 B}{\delta\Gamma^{N_1}\delta\Gamma^{N_2}} + \dots \tag{36}$$

The first term reproduces classical contributions proportional to Poisson bracket of the corresponding classical variables.

In our case the operator $A = \oint(dz/iz)a(z)$ depends on all the canonical variables X^μ, $Y^{+\mu}$, $Y^{-\mu}$ and the operator $B = \oint(dz/iz)b(z)$ depends only on $Y^{+\mu}$ and so the contribution quadratic in contractions $\Delta^{(2)}$ in the commutator is

$$\begin{aligned}
\Delta^{(2)} = \oint\frac{dz_1}{iz_1}\oint\frac{dz_2}{iz_2}\Big\{&2\underbrace{X^{\mu_1}(z_1)Y^{+\nu_1}(z_2)}_A\underbrace{X^{\mu_2}(z_1)Y^{+\nu_2}(z_2)}_S\\
&\qquad\times\frac{\partial^2 a}{\partial X^{\mu_1}\partial X^{\mu_2}}(z_1)\frac{\partial^2 b}{\partial Y^{+\nu_1}\partial Y^{+\nu_2}}(z_2)\\
&+2\Big(\underbrace{X^{\mu_1}(z_1)Y^{+\nu_1}(z_2)}_A\underbrace{Y^{+\mu_2}(z_1)Y^{+\nu_2}(z_2)}_S+\underbrace{X^{\mu_1}(z_1)Y^{+\nu_1}(z_2)}_S\underbrace{Y^{+\mu_2}(z_1)Y^{+\nu_2}(z_2)}_A\Big)\\
&\qquad\times\frac{\partial^2 a}{\partial X^{\mu_1}\partial Y^{+\mu_2}}(z_1)\frac{\partial^2 b}{\partial Y^{+\nu_1}\partial Y^{+\nu_2}}(z_2)\\
&+\underbrace{Y^{+\mu_1}(z_1)Y^{+\nu_1}(z_2)}_A\underbrace{Y^{+\mu_2}(z_1)Y^{+\nu_2}(z_2)}_S\frac{\partial^2 a}{\partial Y^{+\mu_1}\partial Y^{+\mu_2}}(z_1)\frac{\partial^2 b}{\partial Y^{+\nu_1}\partial Y^{+\nu_2}}(z_2)\Big\}.
\end{aligned} \tag{37}$$

Substituting the operators K_n, $L_n^{(0)}$ for A, B in this formula and doing all integrations over z_2 with the use of regularized contractions we arrive at the following commutators:

$$\begin{aligned}
[L_n,L_m] = (n-m)L_{n+m} + \delta_{n,-m}\left(\frac{D}{12}n(n^2-1)+2\alpha(0)n\right)\\
+\int_0^{2\pi}d\sigma\,e^{-i(n+m)\sigma}\Big\{\frac{(m-n)}{16\pi}\Big[e^\gamma(\partial^2+4/\alpha')Q(X)
\end{aligned}$$

$$
\begin{aligned}
&+ e^{-\gamma} Y^{+\mu} Y^{+\nu} Y^{-\lambda} Y^{-\kappa} (\partial^2 - 4/\alpha') F_{\mu\nu\lambda\kappa}(X) \\
&+ \alpha' R^{(2)} Y^{+\mu} Y^{-\nu} \partial^2 W_{\mu\nu}(X) + \alpha'^2 e^{\gamma} R^{(2)} R^{(2)} (\partial^2 + 4/\alpha') C(X) \Big] \\
&+ \frac{i(m^2 - n^2)}{8\pi} \Big[2e^{-\gamma} Y^{+\nu} Y^{-\lambda} Y^{-\kappa} \partial^\mu F_{\mu\nu\lambda\kappa}(X) + \alpha' R^{(2)} Y^{-\nu} \partial^\mu W_{\mu\nu}(X) \Big] \\
&+ \Big[\frac{m - n + n^3 - m^3}{24\pi} + \frac{\alpha(0)(n - m)}{\pi D} \Big] Y^{-\lambda} Y^{-\kappa} F^{\mu}{}_{\mu\lambda\kappa}(X) \Big\},
\end{aligned} \tag{38}
$$

Analogous calculations give the other commutators:

$$
\begin{aligned}
[\bar{L}_n, \bar{L}_m] &= (n - m) \bar{L}_{n+m} + \delta_{n,-m} \left(\frac{D}{12} n(n^2 - 1) + 2\beta(0) n \right) \\
&+ \int_0^{2\pi} d\sigma \, e^{i(n+m)\sigma} \Big\{ \frac{(m - n)}{16\pi} \Big[e^{\gamma} (\partial^2 + 4/\alpha') Q(X) \\
&+ e^{-\gamma} Y^{+\mu} Y^{+\nu} Y^{-\lambda} Y^{-\kappa} (\partial^2 - 4/\alpha') F_{\mu\nu\lambda\kappa}(X) \\
&+ \alpha' R^{(2)} Y^{+\mu} Y^{-\nu} \partial^2 W_{\mu\nu}(X) + \alpha'^2 e^{\gamma} R^{(2)} R^{(2)} (\partial^2 + 4/\alpha') C(X) \Big] \\
&+ \frac{i(m^2 - n^2)}{8\pi} \Big[2e^{-\gamma} Y^{+\mu} Y^{+\nu} Y^{-\kappa} \partial^\lambda F_{\mu\nu\lambda\kappa}(X) + \alpha' R^{(2)} Y^{+\mu} \partial^\nu W_{\mu\nu}(X) \Big] \\
&+ \Big[\frac{m - n + n^3 - m^3}{24\pi} + \frac{\beta(0)(n - m)}{\pi D} \Big] Y^{+\mu} Y^{+\nu} F_{\mu\nu}{}^{\lambda}{}_{\lambda}(X) \Big\}, \\
[L_n, \bar{L}_m] &= \int_0^{2\pi} d\sigma \, e^{i(m-n)\sigma} \Big\{ \frac{(m - n)}{16\pi} \Big[e^{\gamma} (\partial^2 + 4/\alpha') Q(X) \\
&+ e^{-\gamma} Y^{+\mu} Y^{+\nu} Y^{-\lambda} Y^{-\kappa} (\partial^2 - 4/\alpha') F_{\mu\nu\lambda\kappa}(X) \\
&+ \alpha' R^{(2)} Y^{+\mu} Y^{-\nu} \partial^2 W_{\mu\nu}(X) + \alpha'^2 e^{\gamma} R^{(2)} R^{(2)} (\partial^2 + 4/\alpha') C(X) \Big] \\
&- \frac{n^2 + 4\alpha(0)/D}{8\pi} \Big[2e^{-\gamma} Y^{+\nu} Y^{-\lambda} Y^{-\kappa} \partial^\mu F_{\mu\nu\lambda\kappa}(X) + \alpha' R^{(2)} Y^{-\nu} \partial^\mu W_{\mu\nu}(X) \Big] \\
&+ \frac{m^2 + 4\beta(0)/D}{8\pi} \Big[2e^{-\gamma} Y^{+\mu} Y^{+\nu} Y^{-\kappa} \partial^\lambda F_{\mu\nu\lambda\kappa}(X) + \alpha' R^{(2)} Y^{+\mu} \partial^\nu W_{\mu\nu}(X) \Big] \\
&- \left(\frac{n - n^3}{6} - \frac{4n\alpha(0)}{D} + \frac{2\alpha(0)}{D} \right) \frac{1}{4\pi} Y^{-\lambda} Y^{-\kappa} F^{\mu}{}_{\mu\lambda\kappa}(X) \\
&+ \left(\frac{m - m^3}{6} - \frac{4m\beta(0)}{D} + \frac{2\beta(0)}{D} \right) \frac{1}{4\pi} Y^{+\mu} Y^{+\nu} F_{\mu\nu}{}^{\lambda}{}_{\lambda}(X) \\
&- \frac{i}{4\pi\alpha'} \Big[(e^{\gamma})' Q(X) + (e^{-\gamma})' Y^{+\mu} Y^{+\nu} Y^{-\lambda} Y^{-\kappa} F_{\mu\nu\lambda\kappa}(X) \\
&+ \alpha' (R^{(2)})' Y^{+\mu} Y^{-\nu} W_{\mu\nu}(X) + \alpha'^2 (e^{\gamma} R^{(2)} R^{(2)})' C(X) \Big] \Big\},
\end{aligned} \tag{39}
$$

As a result of calculations the parameters Λ, $\bar{\Lambda}$ disappear from the quantum algebra in this approximation. It means that any ordering with various Λ, $\bar{\Lambda}$ corresponds to the same quantum theories just as it is in the case of free string where the

only relevant ordering parameters are $\alpha(0)$ and $\beta(0)$. This is an effect of linear approximation and there should appear dependence on Λ, $\bar{\Lambda}$ at higher levels. The regularization parameter ϵ has cancelled out of the commutators too that is the quantum algebra appeared in the form as if it were finite and did not require any regularization procedure.

Eqs.(39) define the explicit form of the functions $A_{\alpha\beta}$ (4) in the theory under consideration. Ghost contributions $G_{\alpha\beta}$ have the same structure as in free string theory [13] and cancel the $c-$valued terms in (39) provided that $D = 26$ and $\alpha(0) = \beta(0) = 1$.

One can see from (39) that eqs.(8) in our case require $\gamma'(\tau,\sigma) = 0$, $R^{(2)\prime}(\tau,\sigma) = 0$. Together with the condition (14) it means that the operators L_n, $\bar{L}_n$ form conformal algebra only if $\gamma = const$ and so $R^{(2)}(\tau,\sigma) = 0$. This does not contradict the corresponding results of covariant approaches. If effective equations of motion for background fields are fulfilled the quantum effective action obtained by means of functional integration over X^μ does not depend on conformal factor γ. If we write the metric in the form $g_{ab} = e^\gamma \bar{g}^{ab}$ then the effective action depends only on $\bar{g}^{ab}$. But two independent components of $\bar{g}^{ab}$ are constrained by gauge fixing functions. Therefore all physical results (e.g. correlation functions of gauge invariant operators) will be the same for any world sheet including the flat one. So without any loss of generality one can choose γ to be constant and this is the choice that is reproduced within our canonical formulation.

It should be noted that our approach is not restricted by flat world sheets. Quantum theory can be formulated for any functions γ but it is conformal invariant only for constant γ.

If one chooses the conditions $\gamma = 0$, $R^{(0)} = 0$ then the background fields $W(X)$ and $C(X)$ disappear from the classical action (9). Then to fulfill the eqs.(8) the only relevant background fields $Q(X)$ and $F(X)$ should obey the following equations:

$$(\partial^2 + 4/\alpha')Q(X) = 0, \qquad (\partial^2 - 4/\alpha')F_{\mu\nu\lambda\kappa}(X) = 0, \tag{40}$$

$$\partial^\mu F_{\mu\nu\lambda\kappa}(X) = 0, \qquad \partial^\lambda F_{\mu\nu\lambda\kappa}(X) = 0, \tag{41}$$

$$F^\mu{}_{\mu\lambda\kappa}(X) = 0, \qquad F_{\mu\nu}{}^\lambda{}_\lambda(X) = 0. \tag{42}$$

The eqs.(40) are mass-shell conditions for background fields. Other equations show that first massive level is described by a tensor of fourth rank which is symmetrical and traceless in both pairs of indices and transversal in all indices. This exactly corresponds to the closed string spectrum [20] and so our approach gives the full set of correct linear equations for massive background fields.

5 Conclusion

In our paper we proposed a general scheme allowing in certain cases to construct guage invariant quantum formulation starting with a classical theory without gauge

invariance. In usual quantization schemes classical first class constraints are declared operators and they should obey some quantum gauge algebra. Our prescription leads to more general scheme allowing to build the operator of canonical BRST charge Ω generating quantum gauge transformations for a non-gauge classical theory.

We applied the proposed prescription to the theory of closed bosonic string coupled to massive background fields of the first level which is not conformal invariant at the classical level and found conditions of its quantum conformal invariance in linear approximation. Within this approach we derived the full set of effective equations of motion for massive background fields including the tracelessness conditions (42) deriving of which in covariant approaches represents a problem [9, 12].

The obtained results show that our approach can be considered as a general method allowing to construct gauge invariant quantum theories starting from non-gauge classical models. In particular we hope that this method provides a possibility for deriving interacting effective equations of motion for massive and massless background fields within the framework of canonical formulation of string models and provides a justification of covariant functional approach to string theory.

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